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The measurement of fluctuating skin friction in air with heated thin-film gauges

By B. J. BELLHOUSE AND D. L. SCHULTZ
Department of Engineering Science, Oxford University

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A thin-film, heated-element probe is calibrated to measure fluctuating skin friction. The probe is mounted flush with the surface of a flat plate which is oscillated in its own plane, in the direction of the air stream. The relationship between fluctuating heat transfer from the film and fluctuating skin friction is discussed theoretically using Lighthill's (1954) technique.

Introduction

Steady measurements of velocity and skin friction by thin-film techniques are accurate and simple to take, but the measurement of fluctuating velocity in air, by this method, is complicated by thermal feedback from the glass on which the film is deposited (Bellhouse & Schultz 1967). For a wedge-shaped velocity probe the thermal boundary layer produced by the heated film is usually much thicker than the viscous boundary layer on the wedge. At the other extreme lies the problem of a narrow film flush with the surface of a model, set a sufficient distance from the leading edge for the thermal boundary layer, produced by the heated film, to be much thinner than the viscous boundary layer on the model. Liepmann & Skinner (1954) have shown that, if

$$1 \leqslant \frac{\bar{q}_0 L}{k\Theta} < \frac{\sigma}{C_f}, \tag{1}$$

then the thermal layer will lie within the linear region of either a laminar or a turbulent boundary layer. ($\bar{q}_0$ is the mean heat transfer rate from the film to the flow, L the film length in the stream direction, Θ the temperature difference between the film and the flow, C_f the local skin friction coefficient, σ the flow Prandtl number, k the air thermal conductivity.)

Condition (1) was satisfied by the probes used in the experiments described in this paper (for probes of a convenient shape and resistance it is hard not to meet the condition).

Lighthill treated the case of a heated plate, with constant surface temperature, placed in an oscillating flow $U_0 + U_1 e^{i\omega t}$. The viscous and thermal boundary layers started at the same point, and he considered the case of the thermal layer thicker than the viscous layer. However, Lighthill's technique may be applied to the surface thin-film problem, and considerable simplification results from the assumption that the element thermal-layer is thin compared with the plate viscous-layer.

Solution of the thermal and viscous boundary-layer problem

The steady thermal boundary-layer profile over the isothermal heated element is assumed to be of the form

$$T_0 = \Theta(1-\zeta)^3(1+\zeta), \tag{2}$$

where $T = T_0 + T_1 e^{i\omega t}$ is the temperature of the fluid (and varies with position), ζ is a non-dimensional distance, y/θ, where y is the distance normal to the model, and $\theta(x)$ is the thermal boundary-layer thickness.

Since the viscous profile is assumed to be linear for $y \leqslant \theta$, the velocity u_0 may be replaced by $\tau_0 y/\mu$, where $\tau = \tau_0 + \tau_1 e^{i\omega t}$ is the skin friction and μ the fluid viscosity. Substitution of (2) and the linear form of u_0 in the thermal energy integral equation,

$$-\kappa\left(\frac{\partial T_0}{\partial y}\right)_{y=0} = \frac{\partial}{\partial x}\int_0^\theta u_0 T_0 \, dy, \tag{3}$$

where κ is the fluid diffusivity, gives, after integration over the element length,

$$\frac{\bar{q}_0 L}{k\Theta} = \left(\frac{3L^2}{5\mu\kappa}\right)^{\frac{1}{3}} \tau_0^{\frac{1}{3}},$$

where the bar denotes the average of q_0, and

$$\bar{q}_0 = \frac{1}{L}\int_{x_0}^{x_0+L} q_0(x)\, dx = -\frac{9}{4}\frac{k\Theta}{\bar{\theta}}. \tag{4}$$

At low frequencies the fluctuating thermal-profile may be taken to be

$$T_1 = -\Theta B(\omega)\zeta(1-\zeta)^2(1+2\zeta). \tag{5}$$

This satisfies $T_1 = \partial^2 T_1/\partial y^2 = 0$ at $y = 0$ and has a second-order zero at $y = \theta$.

Taking $u_1 = \tau_1 y/\mu$ and using the dynamic form of (3), we obtain the low-frequency solution,

$$q_1(x) = -\frac{k\Theta}{\theta}\frac{\tau_1}{\tau_0}\left[\frac{3}{2} + \frac{1}{15}\frac{i\omega\bar{\theta}^2}{\kappa}\right]^{-1}, \tag{6}$$

which may be averaged to give

$$\frac{\bar{q}_1}{\bar{q}_0} = \frac{1}{3}\frac{\tau_1}{\tau_0}\left[1 + \frac{2}{45}\frac{i\omega\bar{\theta}^2}{\kappa}\right]^{-1}. \tag{7}$$

Lighthill's high-frequency solution needs no modification, since the steady velocity term is irrelevant. This solution is

$$q_1(x) = \frac{iU_1 k}{\omega}\left(\frac{\partial^2 T_0(x)}{\partial x \partial y}\right)_{y=0} \frac{1+2\sqrt{\sigma}}{(1+\sqrt{\sigma})^2}. \tag{8}$$

The effect of heat losses to the probe surround

For thin-film velocity probes used in air, the dynamic calibration is frequency-dependent because of heat exchange between the film and the glass on which it is mounted, despite constant-temperature operation, and the calibration curve is flat from about 200 c/s to near 10 kc/s (Bellhouse & Schultz 1967). The effect will be the same for flush-mounted probes, and a realistic model must make allowances for heat transfer upstream and downstream of the film if the probe is used in air. (This effect should be negligible if the probe is used in water.)

A film of measured length L in the stream direction will heat the glass surrounding it to create an effective length of L_{eff} for steady measurements (L_1 upstream, $L_{\text{eff}} - L_1 - L$ downstream). At frequencies above 200 c/s, the effective length is precisely the measured length, since thermal feedback cannot take place because of the sharp attenuation of the thermal waves. Writing $(\tau_1)_s$ as the fluctuating skin friction derived from the steady calibration, then

$$\frac{1}{3}\frac{(\tau_1)_s}{\tau_0} = \frac{\bar{q}_1 L}{\bar{q}_0 L_{\text{eff}}}. \tag{9}$$

For frequencies above 200 c/s, from equations (6) and (8) with the frequency parameter $\beta \equiv (\omega\bar{\theta}^2/\kappa)^{\frac{1}{2}}$, we obtain

$$\frac{(\tau_1)_s}{\tau_1} = \frac{1}{1+\frac{2}{45}i\beta^2}\left[\frac{(L_1+L)^{\frac{2}{3}} - L_1^{\frac{2}{3}}}{(L_{\text{eff}})^{\frac{2}{3}}}\right] \quad \text{for} \quad \beta \sim 1 \tag{10}$$

and

$$\frac{(\tau_1)_s}{\tau_1} = \frac{25 \cdot 09}{(\sqrt{i}\,\beta)^3}\left[\left(\frac{L_{\text{eff}}}{L_1}\right)^{\frac{1}{3}} - \left(\frac{L_{\text{eff}}}{L_1+L}\right)^{\frac{1}{3}}\right] \quad \text{for} \quad \beta \gg 1. \tag{11}$$

Experimental results

Fluctuating skin friction was obtained by shaking a flat plate (1 in. wide and 1·75 in. long) in its own plane in a steady airstream, effectively modulating the air velocity. The plate was supported by taut wires, and was oscillated electromagnetically. Displacements of up to 0·02 in. were obtained, and these were measured with a capacitance gauge.

A thin-film probe was mounted flush with the flat plate, at a distance of 1·125 in. from the leading edge. Several probes were tried and the most successful, drawn in figure 1, consisted of a 0·006 in. glass strip with the film painted on it, with a 0·001 in. airgap on either side of the strip. The purpose of the airgap was to improve thermal insulation. Even with this modification, the effective length was still four times the measured length, but if the gap had been any larger it might have disturbed the viscous boundary layer.

Measurements of fluctuating heat-transfer from the film to the flow were taken for three mean velocities and nineteen frequencies in the range $212 \leqslant f \leqslant 1181$ c/s, and the data is plotted in figure 2. It was assumed that the frequencies were

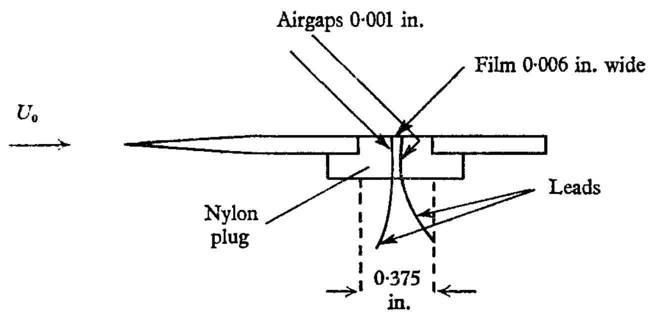

FIGURE 1. Cross-section of calibration plate and thin-film surface-gauge incorporating upstream and downstream airgaps.

high enough for the dynamic calibration to be independent of frequency (although different from the steady calibration), and that for steady measurements the increments in effective length upstream and downstream of the element were roughly equal. The best fit at low frequencies was found if the upstream increment was taken to be $0{\cdot}83L$, and this made the factors in square

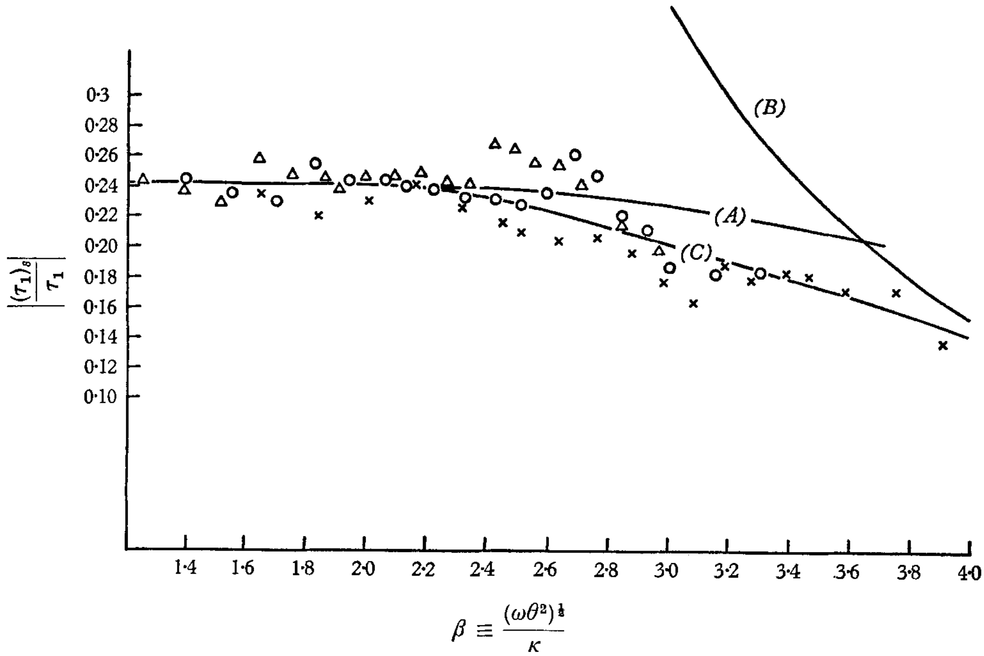

FIGURE 2. The calibration of a surface film to measure fluctuating skin friction: $\times$, $U_0 = 42{\cdot}15$ ft./sec; (A) theory, low frequency; $\bigcirc$, $U_0 = 59{\cdot}43$ ft./sec; (B) theory, high frequency; $\triangle$, $U_0 = 73{\cdot}05$ ft./sec; (C) line fitted to experimental points.

brackets in equations (10) and (11) 0·243 and 0·391 respectively. In figure 2, the curve marked (A) was obtained from (10) and (B) from (11). The data follows curve (C), which is taken to be the actual calibration curve, and which merges smoothly with curves (A) and (B) near $\beta = 2$ and 4 respectively. Much of the scatter may be due to heat transfer upstream of the element, since upstream variations in temperature will affect the fluctuating heat-transfer from the film. Attempts to control the temperature of the surface surrounding the film with another heated element have not yet been successful.

One obvious use of the probe is to measure the spectral density of skin friction in a turbulent boundary layer. While it is true that the response of the probe to fluctuations in skin friction is non-linear, it is permissible to linearize the equation if the fluctuating quantities are much smaller than the mean quantities, for products of frequency components are then negligible. This is normally done for hot-wire measurements in turbulent flows.

It must also be assumed that the laminar sublayer of a turbulent flow is identical with an oscillating laminar boundary layer. It is then possible to apply the dynamic calibration already obtained, if use is made of the mean bridge voltage to calculate the value of $\bar{\theta}$. The root mean square bridge voltage was always less than 4 mV for all frequencies, compared with a mean voltage of about 16 V, so the non-linearity of the equation $V_B^2/\Theta = D\tau_0 + B^{\frac{1}{3}}$ is unimportant.

The spectrum of $10^2|\tau_1/\tau_0|$ on the wall of the low-speed wind tunnel at Oxford for a mean flow velocity of 101·2 ft./sec is shown in figure 3. (The mean velocity profiles were identical to flat-plate profiles.) Also plotted is the spectrum of $10^2|u_1/u_0|$ for a hot-wire placed about 0·005 in. from the surface.

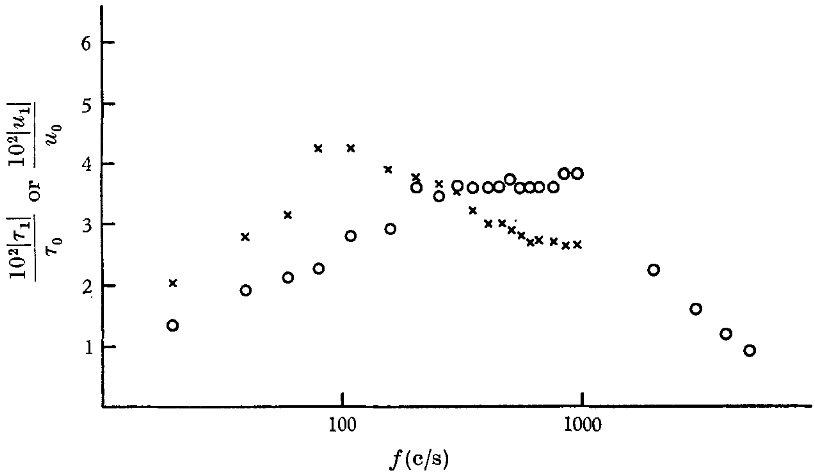

FIGURE 3. Fluctuating velocity and skin friction measurements in a turbulent boundary layer. $U_0 = 101{\cdot}2$ ft./sec; ×, percentage turbulent skin friction, measured with a surface thin film; ○, percentage turbulent velocity, measured with a hot-wire 0·005 in. from the surface.

At frequencies below 200 c/s, the thin-film results will be too high, because thermal feedback has been overcompensated. At very high frequencies, $|u_1/u_0| \neq |\tau_1/\tau_0|$ since the velocity profiles will not be linear, indeed they should approach the shear-wave form

$$u_1/U_1 = (1 - \exp\{-y(i\omega)^{\frac{1}{2}}/\nu\})$$

in which case $$|\tau_1/\tau_0| \gg |u_1/u_0| \quad \text{as} \quad f \to \infty.$$

Conclusion

Lighthill's analysis of fluctuating laminar, viscous and thermal boundary layers on uniformly heated cylinders has been extended to cover the case of a short heated-element embedded in a flat surface. The analysis has been confirmed by experiment, and the possible use of a surface heated-element, in the study of turbulent boundary layers, has been explored.

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